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Nonlinear Three Dimensional Finite Element Analysis of Reinforced Concrete Hollow Beam

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Abstract*: Main aim of this paper is to apply a numerical analysis of reinforced concrete hollow beam using three-dimensional (3D) finite element model under monotonic loading, for this purpose, a well-known finite element traditional software ANSYS has been used. Many variables were studied in the model such as the ratio of hollow area to effective section and the ratio of shear reinforcements. As well as, this study focused on distribution of mesh, density and effects of mesh size on accuracy of finite element model results. Among the two method that representing the cracks, a discrete model has been used. eight-node brick element (SOLID 65) 3D element were used to represent the concrete, while using (LINK 180) 3D finite strain spar element for modeling the flexural and shear reinforcements. The base plates for each loading points and supporting has been ideally obtained by using (SOLID 185) 3D structural solid element. The verification of the finite element model and the materials mechanical properties has been chosen from literature. The advantages of the 3D finite elements model creates a full concept about the model in terms of deformation, stress different directions and critical areas. The importance of this paper will be directed for those who wish to study reinforced concrete hollow beam applications numerically as it provides an idea about how to choose the mesh density and the methods of modeling to achieve accuracy and numerical stability. The results of 3D finite element model shows that the general behavior of the finite element models clarify by the load-deflection curves at mid span and the yielding reinforcement occurrence an appropriate agreement with the compared experimental data from the literature. The maximum tolerance between the modeled sections and its corresponding experimented one were 6.8%.

Keywords: RC hollow beam, three dimension finite element modelling, meshing.

I. INTRODUCTION

This Nowadays, modern facilities trends show an increase in the use of hollow cross section members. This is primarily due to their advantages in stress, torsion and bending response in all directions as well as aesthetic aspects of design as compared to the traditional open-section members. The reinforced concrete hollow section is most widely known for affording economical light weight and long span members [1]. A longitudinal opening is utilized to construct hollow core reinforced concrete members as cast in construction sites, precast and pre-stressed concrete member with voids provided to reduce weight and, the cost of concrete materials and then leads to decreasing the heat induces from hydration [2]. In highly elevated piers and long span members, a hollow section is mostly utilized to reduce the self-weight and development the flexural rigidity of structural members. Moreover the plastic deformation and energy dissipation are also not enough, since it will be create a concrete confinement problem of the webs, so that the shear capacity of the pier will be reduces, this will required a carefully investigation [1].

In addition to piers, the reinforced concrete hollow members have been frequently used in another parts of structure, such as bridge girders, Culverts, and as main spanning. As a girders the hollow section widely used around the world, the bridges on Illinois highway that constructed in 1950s can be the first attempt [3]. Later, this attempt has been extended to be using in many countries. Culverts and some transverse members orthogonal to the main girders on bridges are another application of hollow sections and in addition, pre-cast with or without pre-tensioning or cast-in-place, are used as the main spanning modular construction units [4-6].

Chiad [7] studied the behavior of reinforced concrete hollow members under two points load by tested six concrete members of three solid and the remaining of hollow sections. The main parameters of his work has been the ultimate deformation, which was investigated experimentally and compared with those of the solid members. Also he discussed shear resistant mechanism of these members by focusing mainly on the deterioration of concrete shear resistance, he investigate the effects of the hollow percentages on flexural, shear, and combination between them under monotonic loading, the response of the members are obtained through first cracks load, ultimate loads, crack patterns and load deflection behavior.

Alnuaimi, et al. [8] compare between hollow and solid reinforced concrete members all of members were designed as hollow cross sections to resist combined load of bending, shear and torsion. They were found that the concrete core participates in the members behavior and strength and cannot be ignored when combined load of bending, shear and torsion are present. Its contribution depends partly on the ratio of the torsion to bending moment and the ratio of shear stress due to torsion to the shear stress due to shear force. The longitudinal reinforcements yielded while the shear steel experienced lower strain values. Most of the hollow members was failed near the design loads while the solid failed much more than the design loads. All members failed in a ductile manner with reinforcement yielding before the concrete crushes.

Among the several methods used in the behavior analysis of structures, finite element analysis is widely used based on the nonlinear behavior of materials. The use of finite element analysis software has grown nowadays because of the development in knowledge, high tech and efficiency of computer software packages.

Kotsovos and Spiliopoulos [9] proposed a finite element model of structural concrete also allow for crack closure in localized regions of a structure. This model was used to study the behavior of structural concrete members under many types of loading, encompassing both proportional and sequential loading. The analysis has been found to yield a close fit to experimental values, while neglecting crack closure has a negligible effect on the predicted behavior of structural concrete under proportional loading, sequential loading usually requires a proper allowance of crack closure of sensible analytical predictions are to be achieved.

Sasmal, Kalidoss, and Srinivas [10] discuss nonlinear behavior of reinforced concrete members and fiber reinforced polymer strengthened reinforced concrete members, the certified numerical model was used to assess the load displacement and load strain behavior of the strengthened members. 3D structural solid element SOLID 65 was used to model concrete and LINK 8 truss element is utilized for modeling the reinforcements. The properties of SOLID 65 such as cracking strength, shear transfer coefficients, etc. are extremely sensitive and play a very important role in convergence and accuracy of results.

Madenci and Guven [11] presented and discussed ANSYS software for more complex nonlinear finite element models were assessed, in which longitudinal and shear (stirrups) reinforcements were modeled as built, the same element of Sasmal, Kalidoss, and Srinivas. [10] Were used in this work, the shear transfer coefficient, value of ranges from 0.20 to 0.50, with 0.0 representing a smooth crack makes complete loss of share transfer and 1.0 representing a rough crack no loss of shear transfer. The softening of concrete is occupied into account due to deformation in the models using a failure surface with altered peak compressive and tensile stresses. The stress-strain behavior of concrete in ANSYS software can be defined using Von-Mises or Drucker-Prager yield criterion [12, 13].

This research work utilized the finite element computer software suitable for the nonlinear analysis of reinforced concrete hollow members under increasing loads. Also focuses on 3D finite element simulation technique and compared with experimental data. The computer software takes into consideration the effect of material nonlinearity that results from, tensile cracking, compression crushing of concrete and yielding of reinforcement. Also we study the effect of the opening percentage on whole member behavior in finite element model. The failure criteria of (Willam and Warnke) [14] has been used in this work, because it is able to predicting failure for concrete, in order to modelling the two failure modes cracking and crushing to define a failure surface for the concrete we need to input two parameters ultimate uniaxial tensile strength and compressive strengths. Therefore, a criterion for failure of the concrete due to a multiaxial stress state can be calculated.

II. GEOMETRY OF BEAMS

This study involves modeling solid and hollow reinforced concrete beams with a cross section of 120mm × 180mm and total length of 1000mm, four of these beams contains long opening along the length of different section size. These are 40mm × 40mm and 40mm × 80mm. The flexural reinforcement consisted of 3Ø12mm in lower rebar and 2Ø12mm in upper rebar. The shear reinforcement (stirrups) consisted of two types Ø10mm at 100mm from center to center (c/c) and Ø10mm at 50mm c/c. The beams are analyzed for simply supported condition with a monotonically partial 12 % of total length. Beams Presented and experimentally tested by [2]. The reinforcements and details of test specimens as shown in Fig. 1, and the details of reinforcements as shown in Table 1.

Table 1. Details of the reinforcement [2].

Specimen code	Tension reinforced	Compression reinforced	Stirrups	Hollow ratio
SB 100	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 100 mm	0
SB 50	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 50 mm	0
HB 1	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 50 mm	7.4 %
HB 2	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 50 mm	14.8 %
HB 3	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 100 mm	7.4 %
HB 4	3 Ø 12 mm	2 Ø 12 mm	Ø 10 @ 100 mm	14.8 %

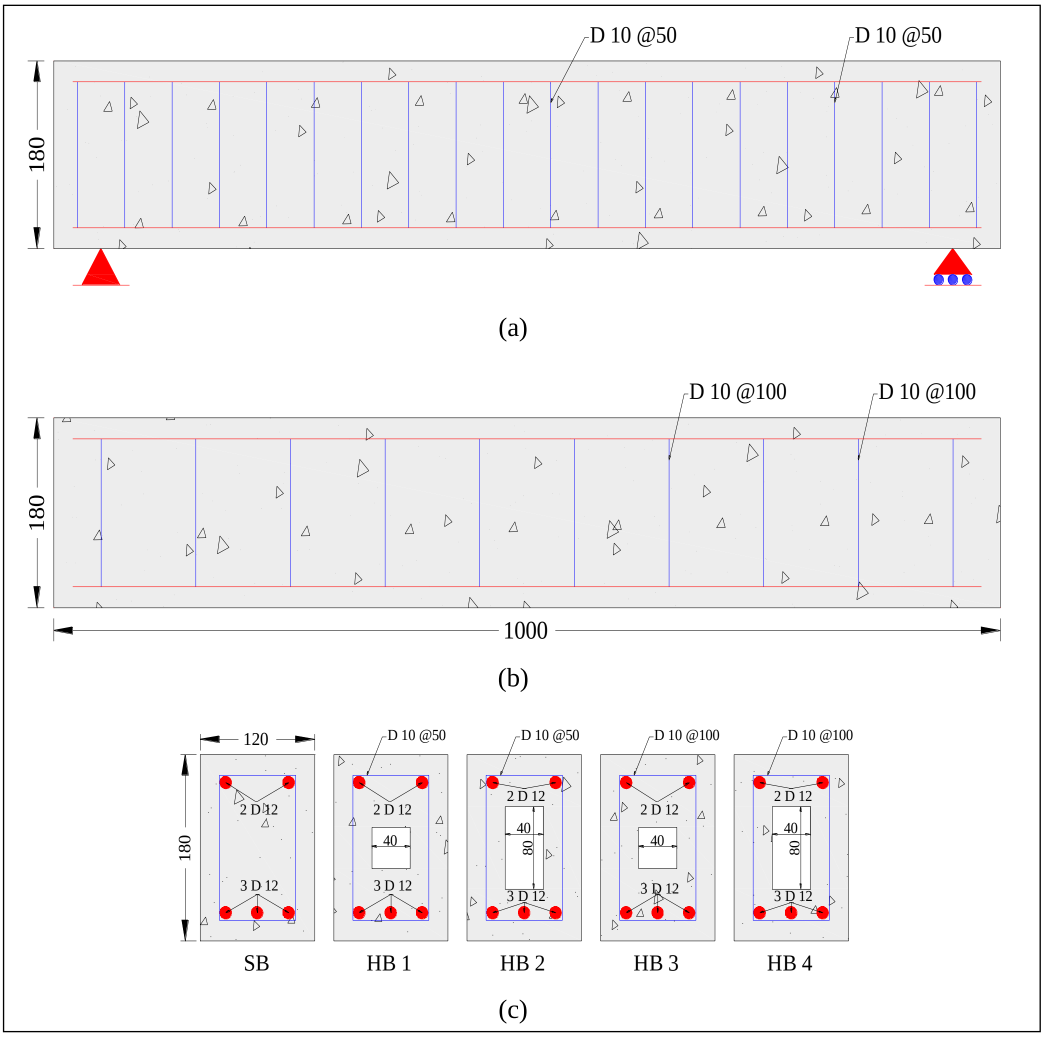
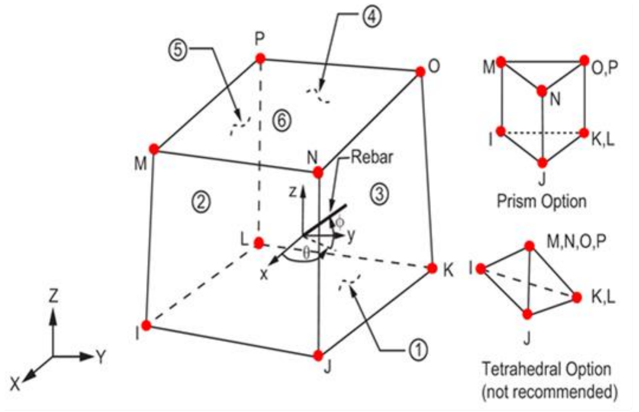


Fig. 1. Details of specimen, longitudinal and shear reinforcement: (a) Shear reinforcement Ø10mm at 100 mm, (b) Shear reinforcement Ø10 mm at 50mm, (c) cross section. All units in mm [2].

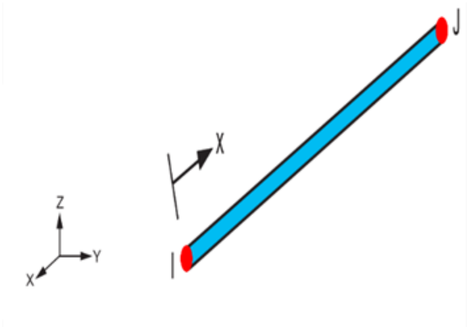
III. FINITE ELEMENT MODELING

A. Type of elements

The concrete has been modeled using SOLID 65 element as shown in Fig. 2, (a) .The solid element has 3D eight nodes brick elements, with three degrees of freedom translation in the (X, Y and Z direction). This element is suitable for representing the behavior of concrete in two different state, cracking in tension and crushing in compression. The reinforced concrete is capable of cracking (in three orthogonal directions), crushing, plastic deformation and creep. In different cases for which the element is likewise applicable would be reinforced composites like fiberglass, steel fiber and geological materials (such as rock) [12].



(a)



(b)

Fig. 2. (a) Geometry of SOLID 65 element [12], (b) LINK180 3D spar geometry [12].

The flexural reinforcement and shear reinforcement (stirrups) were modeling by using LINK180 3D spar element is a uniaxial tension-compression element with three degrees of freedom at each node (translations of the nodes in X, Y, and Z-directions), this element is also capable of plastic deformation. The geometry node location and coordinate system for this element are shown in Fig. 2, (b) [12].

The SOLID 185 3D structural solid element is assign with eight nodes having three degrees of freedom at each node, (translations in X, Y and Z directions). SOLID 185 structural solid is convenient for modeling general 3D solid structures. Steel plate is added under point load (applied load) and supports in order to avoid stress concentration problems. As shown in Fig. 3, [12].

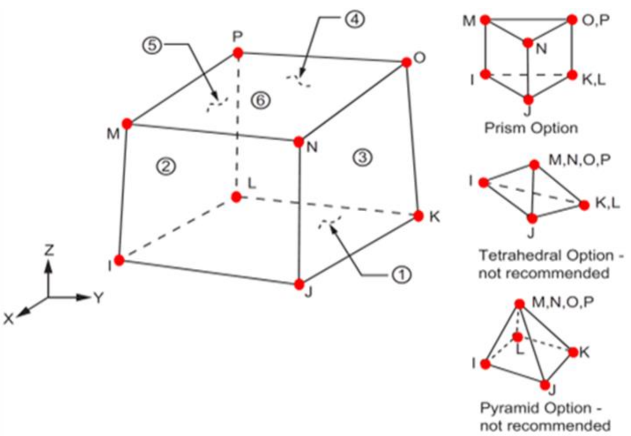


Fig. 3. Geometry of SOLID 185 element [12].

*B. **Material properties***

Concrete is a quasi-brittle material and has a different behavior in both compression and tension. The behavior of reinforced concrete in the numerical model is a real challenge. The nonlinear behavior attributed to the figuration and gradual growth of micro cracks under monotonic loading. The tensile strength of concrete is usually 8-15% of the compressive strength. Fig. 4, shows a typical stress-strain curve for normal weight concrete [15].

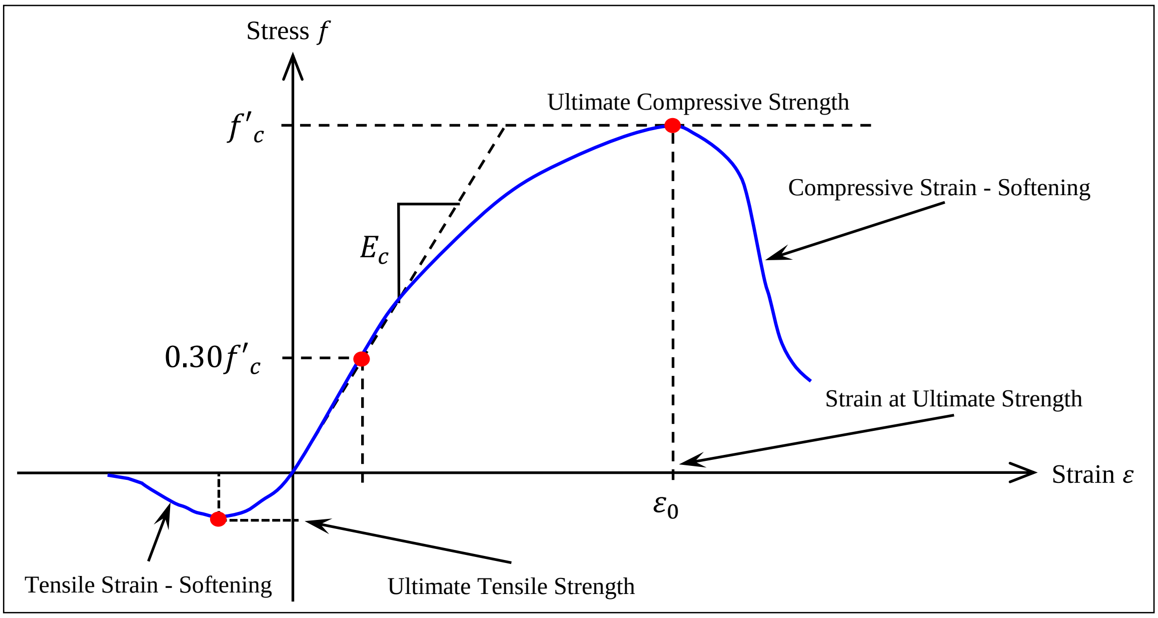


Fig. 4. Typical stress-strain curve for normal weight concrete [15].

In the compression, the stress-strain curve for concrete is linearly elastic up to about thirty percent of the compressive strength. While, above this point, stress increases gradually up to the maximum compressive strength, after that it reaches the compressive strength, the curve slump into a softening region, and eventually the crushing failure occurs at an ultimate strain. In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. Finally in this point, the concrete cracks and the strength decreases gradually to the zero [14, 15].

Various mathematical models are available to approximate this nonlinear behavior to the concrete. In present research work, the uniaxial compressive stress-strain relationship for the concrete is obtained using following equations to compute the stress-strain curve. Fig. 5, the stress- strain relationship for the concrete model was obtained by using the following equations to compute the multilinear isotropic stress- strain curve for the concrete [17].

$$f = \frac{E_c \varepsilon}{1 + (\frac{\varepsilon}{\varepsilon_0})^2} \quad (1)$$

$$\varepsilon_0 = \frac{2 f'_c}{E_c} \quad (2)$$

$$E_c = \frac{f}{\varepsilon} \quad (3)$$

$$E = \frac{f}{\varepsilon} \quad (4)$$

Where:

f = stress at any strain (MPa).

ε = strain at stress f (mm/mm).

ε_0 = strain at peck point.

f'_c = the ultimate compressive strength of concrete in (MPa).

E_c = the modulus of elasticity of concrete in (MPa).

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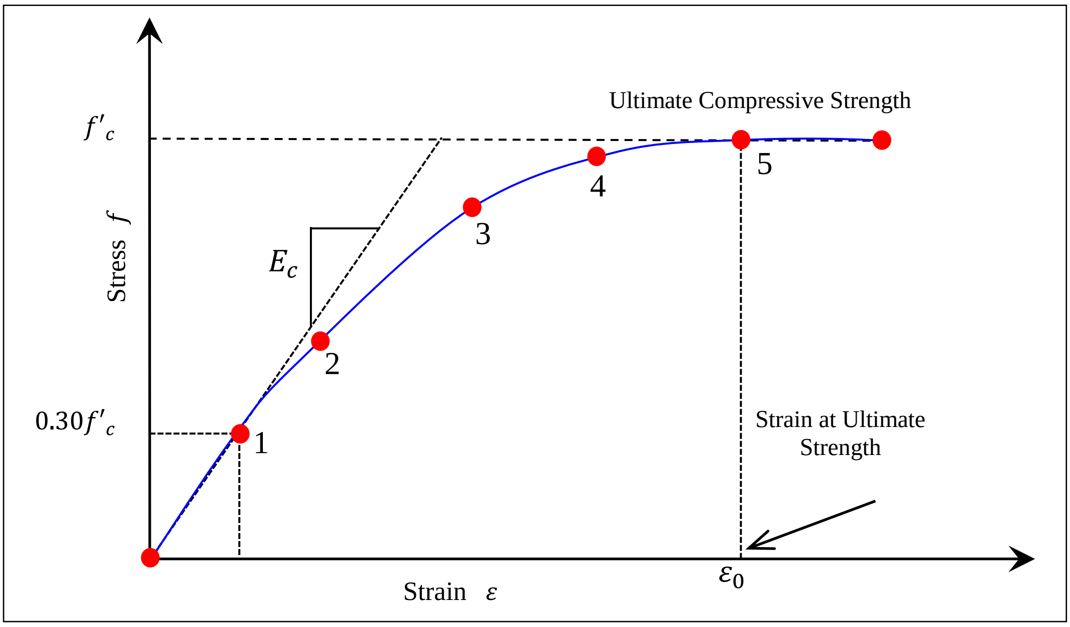

Fig. 5. Typical uniaxial compressive and tensile stress-strain curve for concrete.

Steel reinforcement the mechanical behavior of reinforcing steel bar is assumed to be elastic perfectly plastic material, identical in tension and compression in modeling of steel as shown in Fig. 6,. For incorporating steel material model, the indispensable inputs are modulus of elasticity, tangent modulus and the yield strength [15]. The properties of material is tabulated in Table 2.

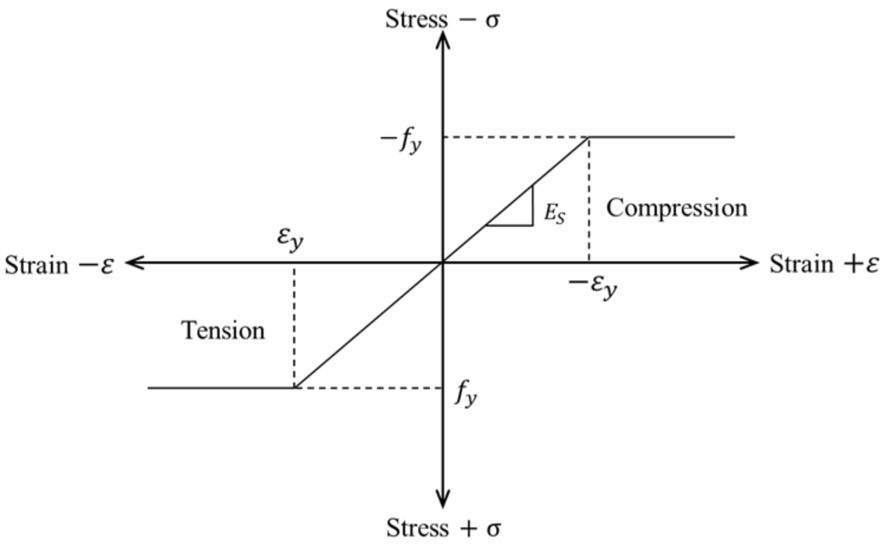

Fig. 6. Strain curve for the steel reinforcement.

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Table 2. Material properties used in the model.

Component	Material property	Values
Concrete	Modulus of elasticity	25100 MPa
	Tensile strength	3.31 MPa
	Poisson's ratio	0.2
	Open shear coefficient	0.3
	Closed shear coefficient	0.95
Reinforcements	Modulus of elasticity	200000 MPa
	Poisson's ratio	0.3
	Yield stress flexural	480 MPa
	Yield stress shear (stirrups)	421 MPa
Steel plate (supports and loading plate)	Modulus of elasticity	200000 MPa
	Poisson's ratio	0.3

Poisson's ratio υ for concrete was assumed to be 0.2.The strength of concrete in tension is typically (8% - 15%) of the compressive strength [18]. The shear transfer coefficients for open and closed cracks were taken as 0.3 and 1 based on the work of previous researches [15,18,19] and turn off the crushing capability of the concrete element as suggested by past [20,21]. A Poisson's ratio of 0.3 is used for the steel reinforcement. The steel plates were modelled as linear isotropic materials with modulus of elasticity 200000 MPa and Poisson's ratio 0.3.

C. Sectional properties (real constant)

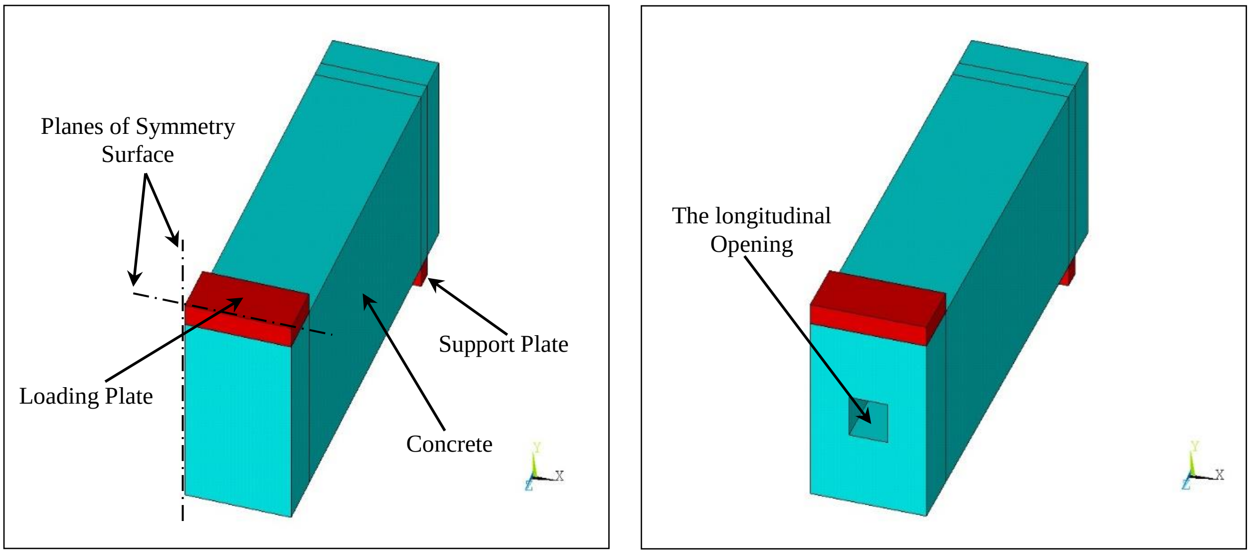
Real constants are properties that depend on the element type such as cross sectional properties of element. In this study, no real constant set exists for the SOLID 185 element. Real constant set 1 is used for reinforced concrete SOLID 65 element. Real constants sets 2 and 3 are defined for the reinforced LINK 180, values for the cross sectional area and in initial condition strain were entered. Cross-sectional areas in set 2 and 3 refer to the longitudinal reinforcement and the shear reinforcement respectively [12]. A value of zero was entered for the initial strain because there is no stress in initial condition at the reinforcement. The real constants details shown in Table 3.

Table 3. The real constants details.

Material details	Element types	Real constants	
Concrete	SOLID 65	N/A	
Longitudinal reinforcement	LINK 180	Cross- Section area (mm2)	113.09
		Initial strain (mm/mm)	0
The shear reinforcement (stirrups)	LINK 180	Cross- Section area (mm2)	78.5
		Initial strain (mm/mm)	0
Steel plate (supports and loading plate)	SOLID 185	N/A	N/A

D. Modeling of the section

The concrete beams, steel plate supports and loading steel plate were modeled as volumes. Take advantage of the model symmetry, reduce computational time and computer memory, half of the beam was modeled in the finite element software by using the appropriate boundary conditions due to symmetry, reflect about XY plane . The combined volumes of the concrete beam, steel plate supports and loading steel plate for solid and hollow beams are shown in Fig. 7, (a, b) respectively.

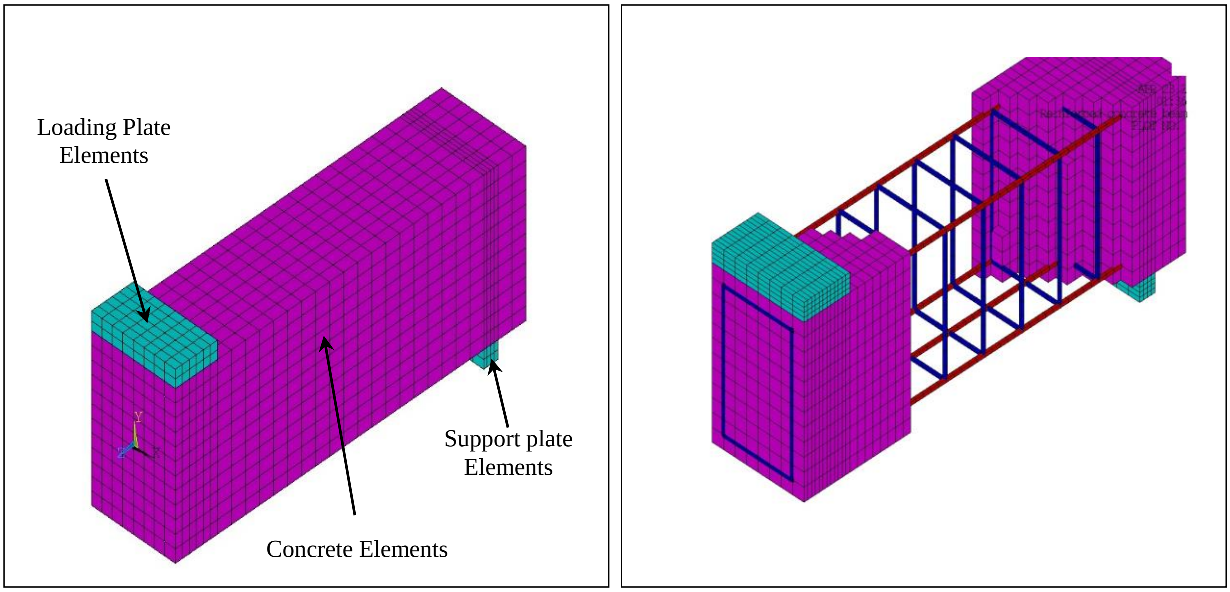


(a) (b)

Fig. 7. Volumes of half beam created in software, (a) Solid, (b) Hollow.

E. Meshing

After creating of volumes, a finite element analysis requires meshing of the model. The concrete beam was meshed such that it consisted of square or rectangular elements. To obtain good results from the concrete element SOLID 65, the use of a rectangular mesh is recommended [19, 20]. The hollow sections is divided to a uniform part in order to obtain the rectangular mesh and reduce the irregular shape of meshing, the volume sweep command was used to mesh the steel plate and support meshing of 3D model as shown in Fig.8,. Convergence study was carried out to determine an appropriate mesh density. The graph was plotted between the different numbers of elements and the mid span deflection for the particular load. Fig. 9 shows the convergence study of the beam.



(a) (b)

Fig. 8 (a) Meshing of the concrete beam, steel plate, steel support, (b) Reinforcement details.

The discrete representation is used to modelling the steel reinforcements by using the (3D spar Link 180 element), the reinforcement in the discrete model uses (3D spar Link 180 element) that are linked to concrete mesh nodes. Therefore, the concrete and the reinforcement mesh share the same nodes and concrete occupies the same regions occupied by the reinforcement as shown in Fig. 10, [21]. In this work, the concrete and the reinforcements are discretized into elements with the same geometrical boundaries and the effects of reinforcing are averaged within the related element, by using symmetry expansion in software full 3D model was presented. The isometric view of full model, reinforcement configuration and concrete element with discrete reinforcement as shown in Fig. 11, (a), (b), (c) respectively.

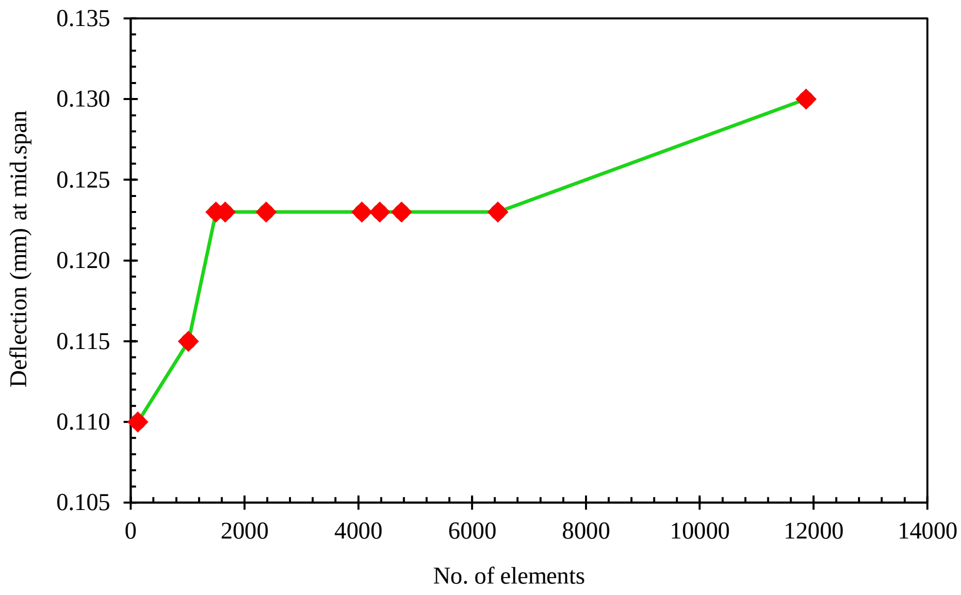


Fig. 9. Convergence study of the model.

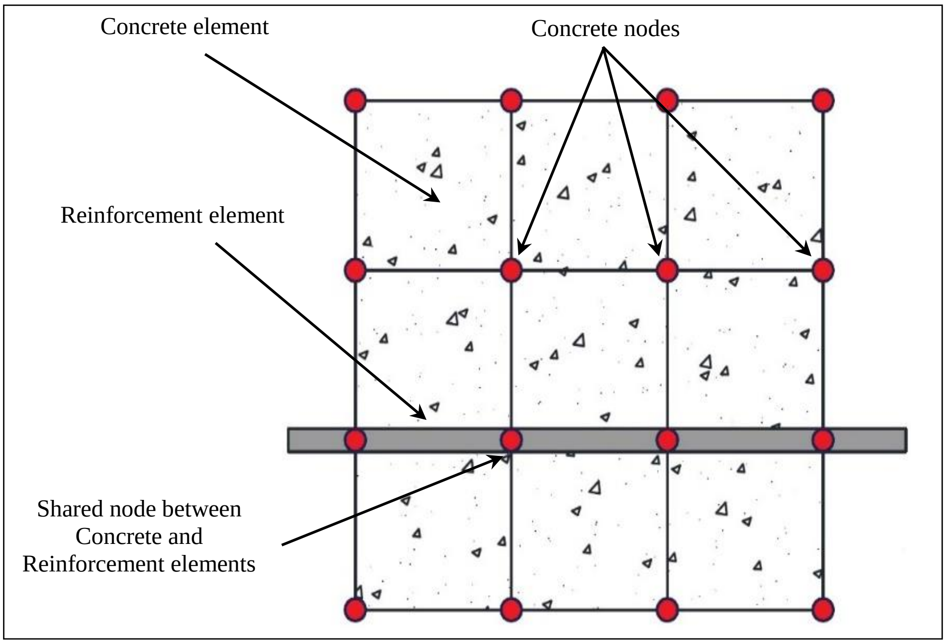


Fig. 10. The discrete model.

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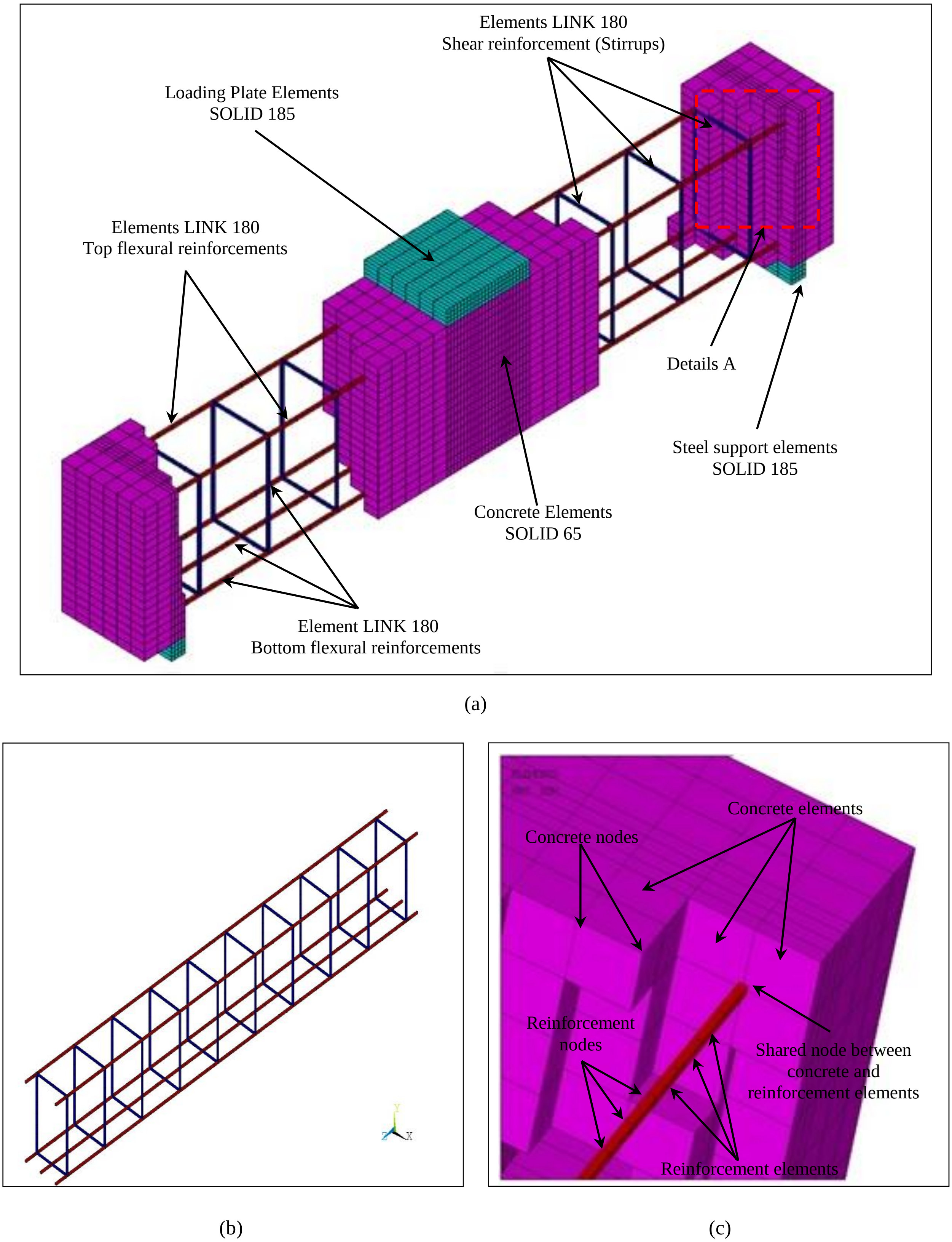
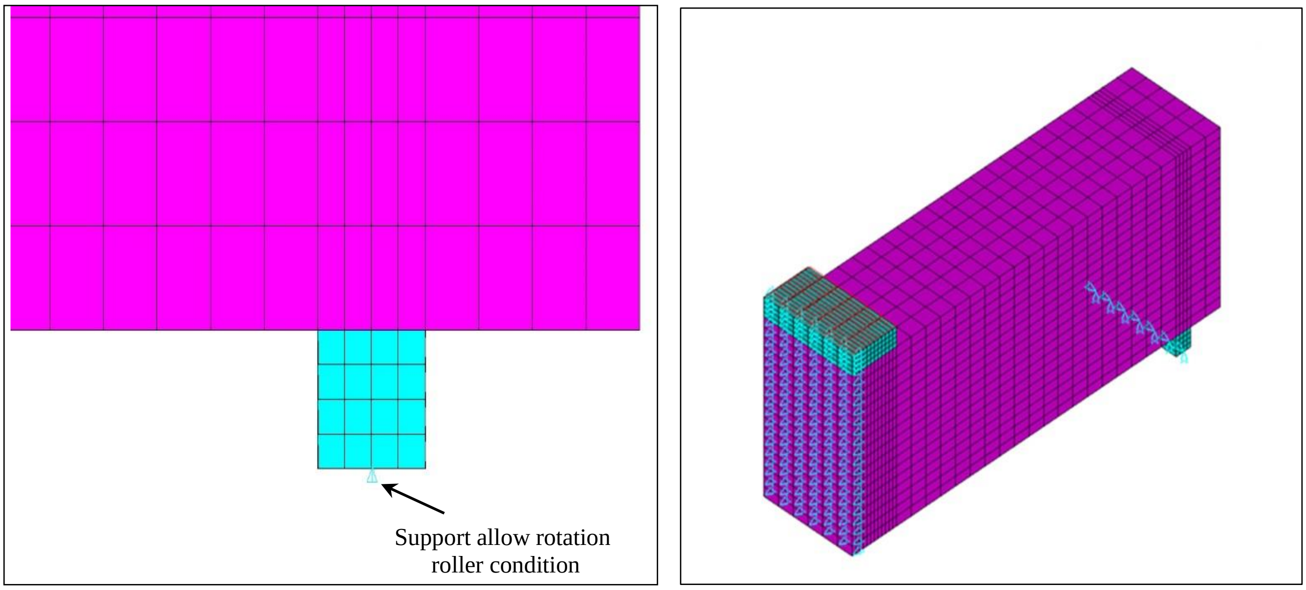


Fig. 11. (a) The isometric view of full Model, (b) Reinforcement modeling, (c) Details A concrete element with discrete reinforcement.

F. ***Loading and boundary conditions.***

The external monotonic loading was applied on a steel plate distributed in the top surface area of the steel plate at the center of beam. As a result, the uniformly distributed load was represented by the equivalent pressure on the top surface area of the plate, simply supported boundary conditions are applied to the beam, single line of nodes in the X axis at the bottom of the steel support was given displacement constant in the Y and X directions, allowed rotation and sliding to simulate the roller condition, to model the symmetry boundary conditions, Select all nodes on the XY plane and restricted. These nodes were given the constraint for three degree of freedom. Loading and boundary conditions as shown in Fig. 12.



(a) (b)

Fig. 12. Loading and boundary conditions (a) Support, (b) Loading and symmetry.

IV. RESULTS AND VALIDATION OF PROPOSED MODELS

The results obtained from the numerical models accomplished by using ANSYS software are compared with the experimental results obtained from the laboratory test. Both solid and hollow beam results are validated with experimental laboratory results by means of load vs. displacement curvy, the comparison it has been tabulated in Table 4.

Table 4. Comparison between FE model and experimental.

Symbol	Ultimate Load (N)		Centerline Deflection (mm)	
	Experimental	Numerical	Experimental	Numerical
SB100	60000	64400	3.320	3.489
SB50	87500	91420	3.720	3.990
HB1	55000	57000	8.080	8.398
HB2	35000	37000	5.699	6.180
HB3	40000	41000	5.700	5.885
HB4	27000	29000	4.300	4.475

The maximum deflection was observed at mid-span at the center of the bottom face of the beams. The results from numerical model were compared with the experimental data, are illustrated in Fig. 13, to Fig. 15, The load deflection curve shows excellent agreement in numerical model with the experimental results throughout the entire range of behavior and failure mode, for numerical models deflection values was measured at the mid –span nodes to avoid disturbance the results due to cracking and crushing of the concrete elements, for all beams numerical model is stiffer than the experimental beams. Fig. 16, Deflections in numerical model, Fig. 17, Reinforcements yielding for beam HB 4 and Stresses flow for beam HB 4 given in Fig. 18,.

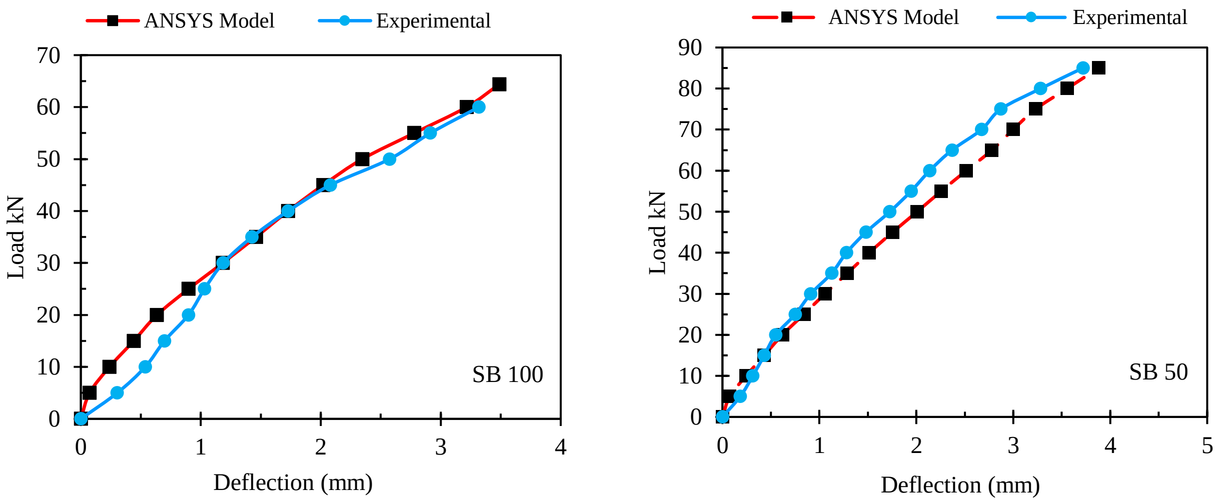


Fig. 13. Comparison of deflections between experimental and numerical model for solid beam.

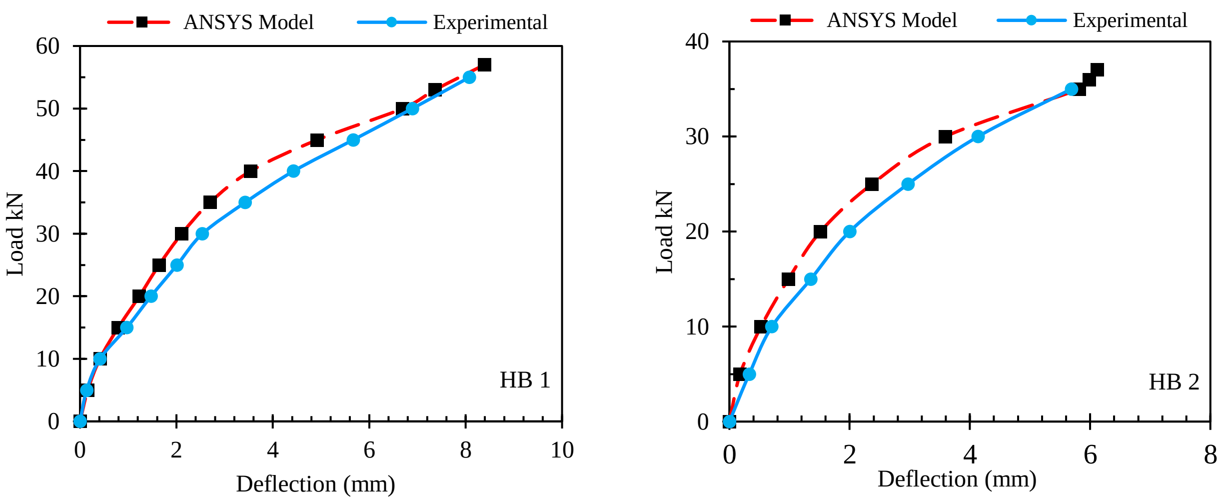


Fig. 14. Comparison of deflections between experimental and numerical model for hollow beams.

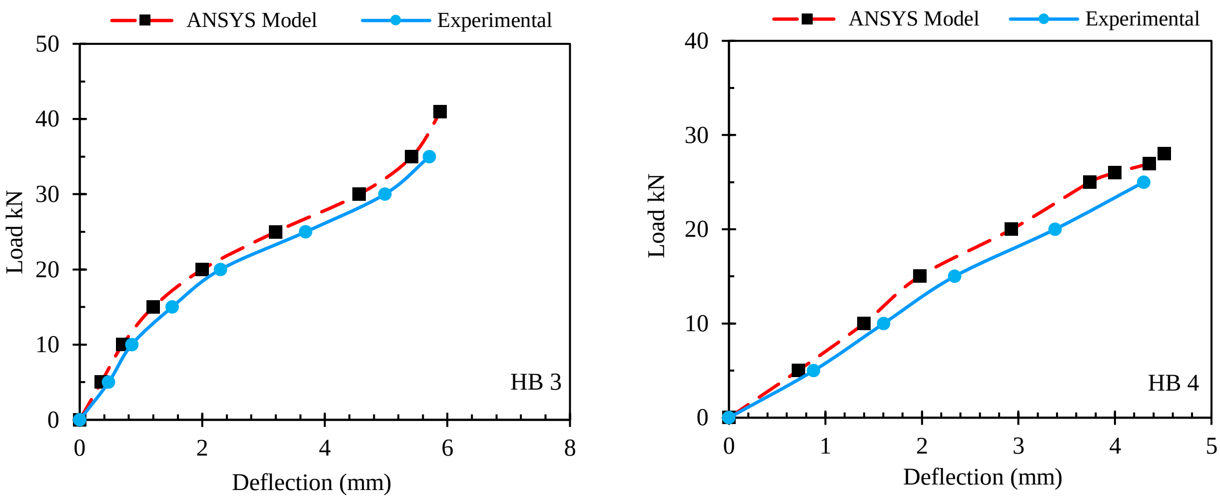
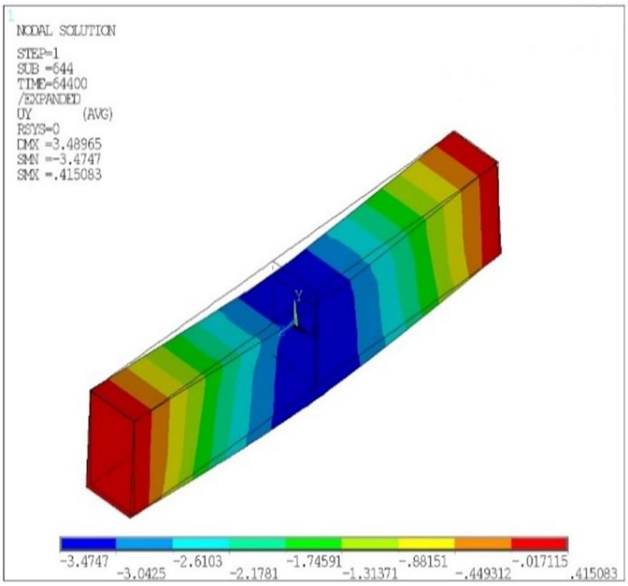
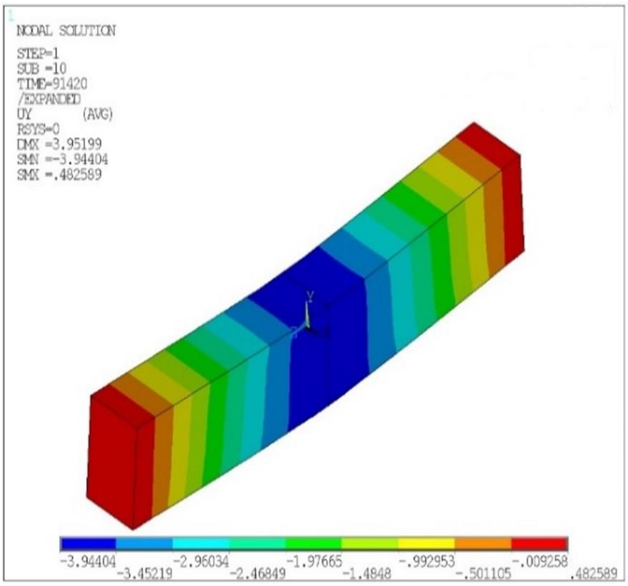


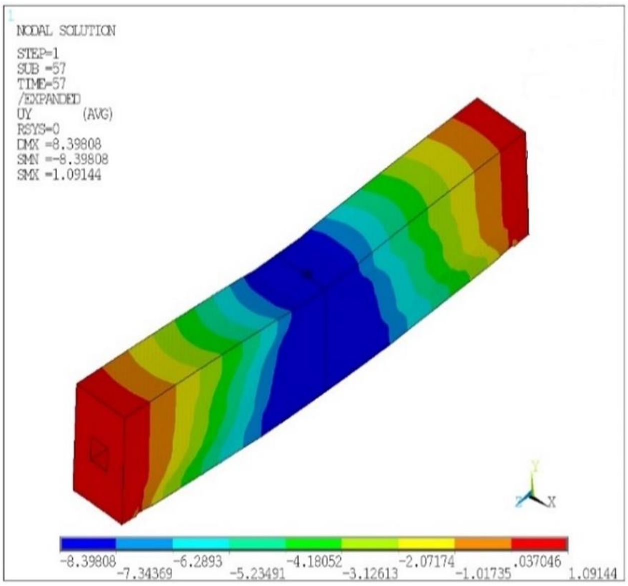
Fig. 15. Comparison of deflections between experimental and numerical model for hollow beams.



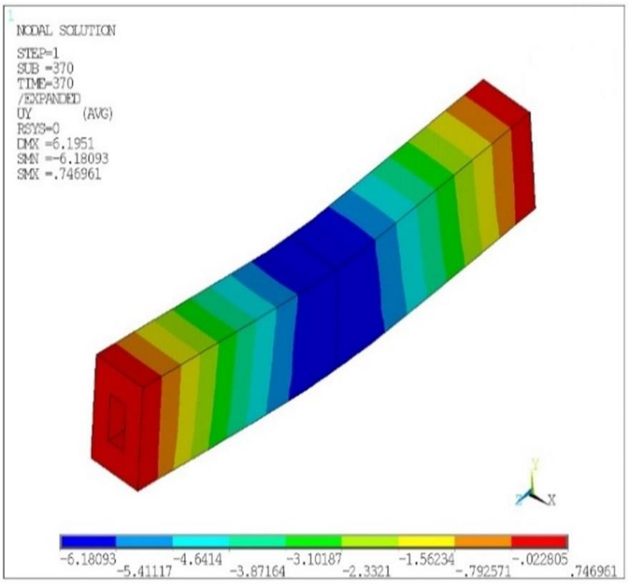
A. (SB 100)



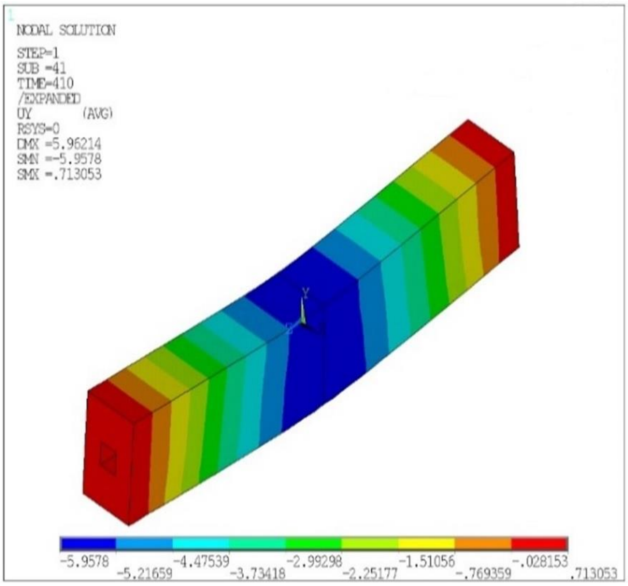
B. (SB 50)



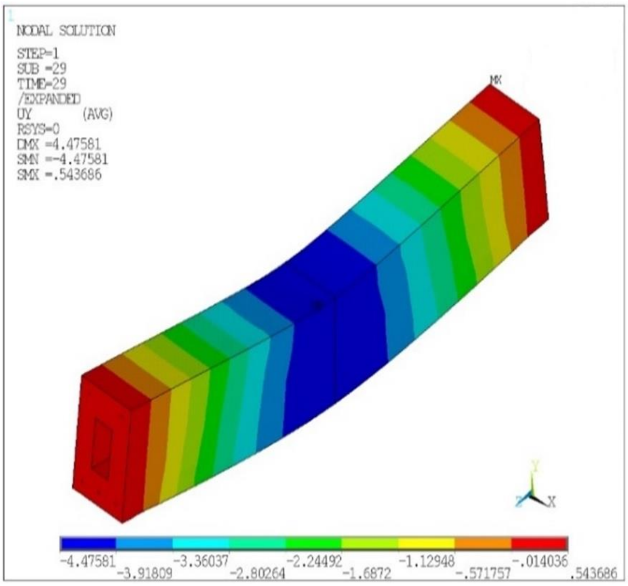
C. (HB 1)



D. (HB 2)



E. (HB 3)



F. (HB 4)

Fig. 16. Deflections in numerical model.

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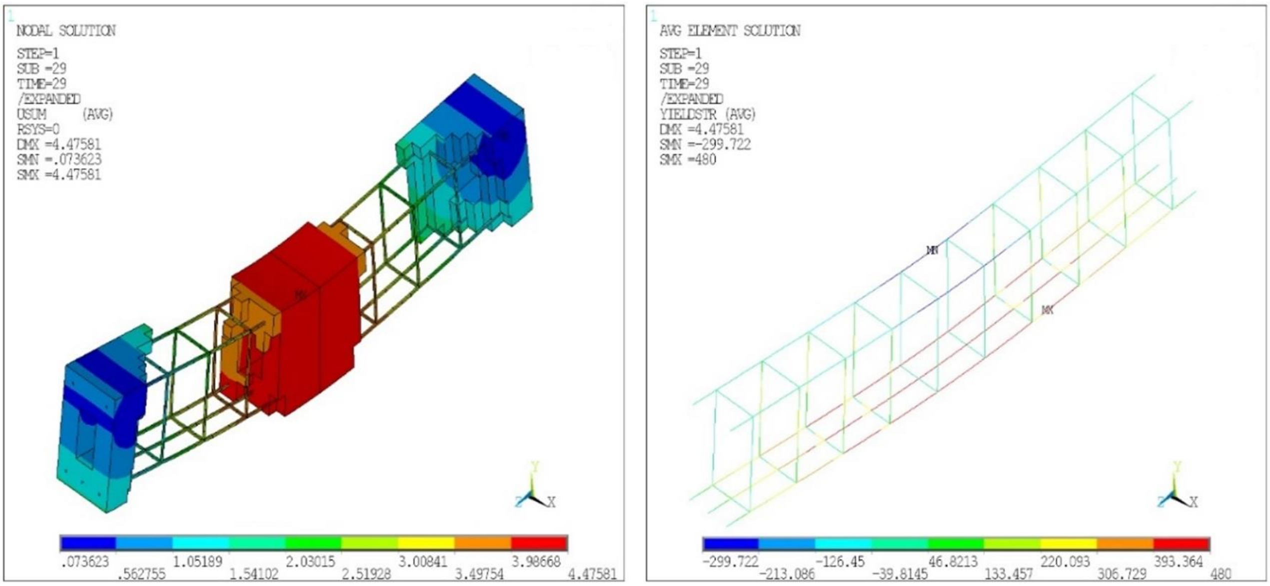

Fig. 17. Reinforcements yielding for beam HB 4.

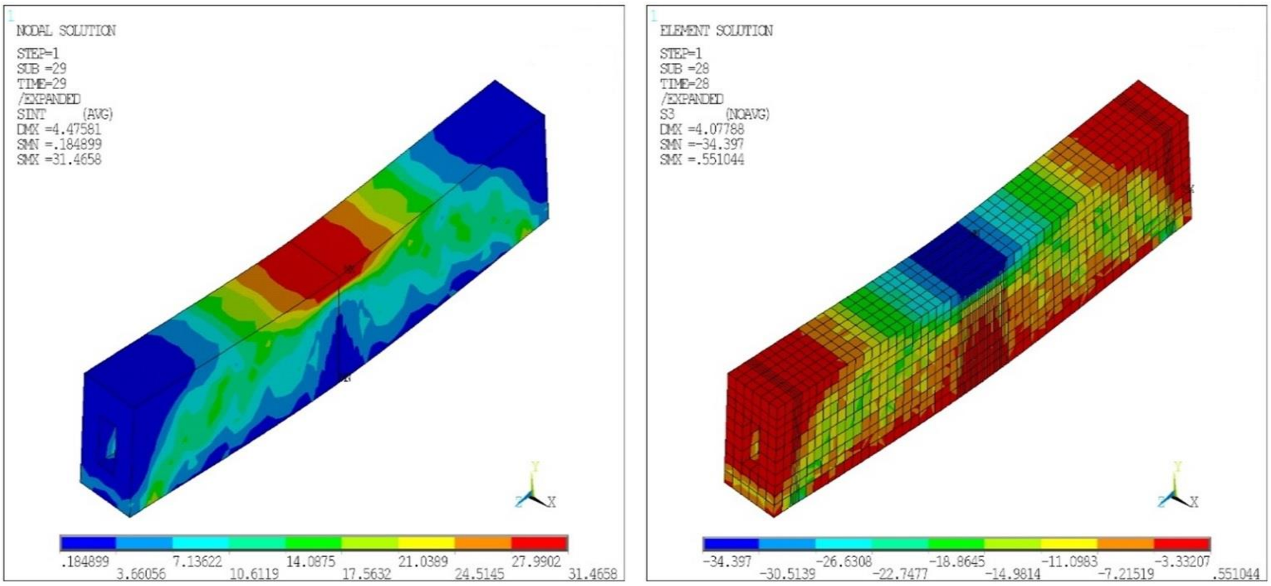

Fig. 18. Stresses flow in hollow beam HB 4.

V. CONCLUSIONS

This study presented a novel modelling for the analysis of reinforced concrete hollow beams in the finite element ANSYS software. The effect of the partial interaction problem has been considered using the principle of finite element. The 3D nonlinear finite element model used in the present paper is capable of simulating the behavior of reinforced concrete hollow beam. The finite element results are in acceptable agreement with the corresponding experimental results. A study was carried out on the effects of the hole in the reinforced concrete finite element model. It was demonstrated that, the hole effect on mesh distribution and therefore affect the stresses, for which the partial interaction effects are significant and must be taken into account. It was found that all solid and hollow beams cracked at higher loads than the load in experimental paper. This paper showed the use of a family of interface ANSYS elements Library which, combined with appropriate finite elements allowed the numerical simulation of hollow core reinforced concrete beam. The modeling of concrete by eight-node brick elements SOLID 65, the steel reinforcement by discrete axial elements LINK 180, gives good accuracy in comparison with experimental results. The failure loads predicted were very close to the failure loads measured during experimental testing. The maximum difference in ultimate loads for were 6.8%.

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