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SLIDING MODE CHATTERING CONTROL AND APPLICATION

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Abstract: ***This paper focuses mainly on the classical variable structure control (VSC), also known as sliding mode control (SMC) theory briefly In section1.2.the paper introduces the classical principle of sliding mode control method. Standard sliding modes provide for finite-time convergence, precise keeping of the constraint and robustness with respect to internal and external disturbances are explained by example.1.However, often they also exhibit a serious drawback which essentially hinders their practical applications. This drawback is high frequency oscillations which inevitably appear in any real system is usually called chattering. This is a highly undesirable phenomenon, because it causes serious wear and tear on the actuator components. Therefore, two methods to eliminate chattering have been proposed in this paper. In section 3.1, Boundary Layer method is proposed and explained. In section 3.2 Asymptotic Sliding Mode approach is proposed and explained with example. 2. Conclusions are stated and further work on chattering elimination are listed***

Keywords: ***variable structure control, Sliding mode control, constraint, robustness, chattering, Boundary Layer.***

I. INTRODUCTION

1.1 Sliding Mode Control

Sliding mode control is known to yield very good robustness properties and is one of the most popular approaches to nonlinear tracking/system control. The inherent robustness to modeling errors of sliding mode controllers has inspired design approaches based on simplified 3rd-order models where the pressure states has been replaced by one single pressure state Also the impressive robustness properties of VSC was fully recognized during this period leading to more interest from the general research community. Applications in a variety of engineering challenges also helped the VSC technique's popularity.

1.2. IDEAL SLIDING MODE CONTROLLER DESIGN

As described by Utkin (1977) the basic concept of VSCs is that the system is allowed to change structure at any instant. The design problem in VSCs is therefore to find the parameters in each of the system structures and to design the switching logic which decides when the structure of the system should change. One of the key features of Variable Structure Systems is that the resulting system can show properties not present in any of the separate system structures. For instance a desired trajectory in the phase plane can be constructed from parts of the trajectories of the separate structures. The motion described by these trajectories is known as *sliding mode*.

The response of a system with sliding mode control can basically be divided into two parts

1. *The reaching phase*: When $s \neq \mathbf{0}$, the system is said to be in the reaching phase. The trajectories in the phase plane will in this phase move toward s.
2. *The sliding phase*: Once the system reaches s the trajectories will move along the surface described by s until it reaches the equilibrium point.

To illustrate some of the concepts and properties of the VSC with sliding mode control mentioned earlier a simple example is introduced where a second-order non-linear system is used as the plant for the control design.

Example.1

Considering the second order system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x) + g(x)\,u + \delta(t) \quad (1.1)$$

where f(x) and g(x) are known (generally nonlinear) functions and δ(t) represents any bounded uncertainties, such as modeling imperfections and bounded disturbances.

The sliding manifold, s could e.g. be defined by $s(x) = x_2 + a_1 x_1 = 0$. This yields the following system behavior once the system has reached the manifold system behavior once the system has reached the manifold

$$x_2 = -a_1 x_1 \quad \Longrightarrow \quad \dot{x}_1 = -a_1 x_1$$

which implies that both x1 and x2 converges exponentially to zero.

The next step is to define a control law that ensures that the sliding manifold is reached in finite time and remains there for all future time. First the time derivative of the sliding function s(x) is found to be

$\dot{s} = a_1x_2+\dot{x}_2 = a_1x_2+f(x) + g(x)u+ \delta(t)$

Suppose that δ (t) satisfies the inequality

$|\delta| \leq D$. By defining the Lyapunov-like function candidate

$$V = \frac{1}{2}s^2 \qquad (1.2)$$

In order to provide the asymptotic stability of Eq. (1.2) about the equilibrium points**s**= 0, the following conditions must be satisfied:

(a) $\dot{V} < 0$ for $\mathbf{s} \neq 0$

(b) $\lim_{|\sigma|\to\infty} V = \infty$

Condition (b) is obviously satisfied by V in Eq. (1.2). In order to achieve finite-time convergence (global finite-time stability), condition (a) can be modified to be

$$\dot{V} \leq -\alpha V^{\frac{1}{2}}, \quad \alpha > 0 \qquad (1.3)$$

Indeed, separating variables and integrating inequality (1.8) over the time interval

$0 \leq \tau \leq t$, we obtain

$$V^{\frac{1}{2}}(t) \leq -\frac{1}{2}\alpha t + V^{\frac{1}{2}}(0) \qquad (1.4)$$

Consequently, V (t) reaches zero in a finite time t_r that is bounded by

$$t_r \leq \frac{2V^{\frac{1}{2}}(0)}{\alpha} \qquad (1.5)$$

Therefore, a control u that is computed to satisfy Eq. (1.3) will drive the variable α to zero in finite time and will keep it at zero thereafter. The derivative of V is computed as

$\dot{V} = s\dot{s} = s[a_1x_1+f(x)] + s\,\delta(t)+g(x)\,s\,u \quad (1.6)$

$\leq s[a_1x_1 + f(x) + s\,\delta(t) + g(x)]\,u$

Assuming $g(x)u = -cx_2 + v$ and substituting it into Eq. (1.6) we obtain

$\dot{V} = s\,\dot{s} = s[a_1x_1 + f(x)] + s\,\delta(t) + g(x)\,s\,u$

$\dot{V} = s[f(x) + s\,\delta(t) + v] = |s|L + |s|D + sv$

(1.7)

Selecting $v = -\rho\, sign(s), \quad \rho > 0$

$$\text{sign}(s) = \begin{cases} 1 & if\ x > 0 \\ -1 & if\ x < 0 \end{cases} \qquad (1.8)$$

And sign (0) $\in [-1, 1]$ (1.9)

with $\rho > 0$ and substituting it into Eq. (1.7) we obtain

$$\dot{V} = |s|L + |s|D + s(-\rho\, sign(s))$$
$$= |s|L + |s|D - \rho\, s\, sign(s)$$
$$= |s|L + |s|D - \rho\, s\frac{|s|}{s}$$
$$= -|s|(\rho - L - D)$$

Since $\dot{V} \leq -\alpha V^{\frac{1}{2}} = \alpha\left(\left(\frac{1}{2}s^2\right)^{1/2}\right.$

$$\leq -\frac{\alpha}{\sqrt{2}}|s| = -|s|(\rho - L - D)$$

Therefore $\rho = L + D + \frac{\alpha}{\sqrt{2}}$ (1.10)

Consequently a control law u that drives s to zero in finite time (1.5) is

$g(x)\,u = -cx_2 - \rho\, sign(s)$ (1.11)

This means that all trajectories of the system reaches the sliding manifold S in finite time and will remain there for all future time.

Remark1.1. It is obvious that $\dot{\sigma}$ must be a function of control u in order to successfully design the controller in Eqn. (1.11).This observation must be taken into account while designing the variable s**.**

Remark1.2. The two components of the control gain Eq. (1.10) is designed to compensate for the bounded disturbance f (x) and $\delta(t)$ while the second term $\frac{\alpha}{\sqrt{2}}$ is responsible for determining the sliding surface reaching time given by Eq. (1.5). The larger the α shorter, the reaching time.

Figure 1.1 shows the responses of the ideal controller with $f(x) = x_1^2 + \sin(x_2)$ and $\delta(t) = \sin(t)$. The controller

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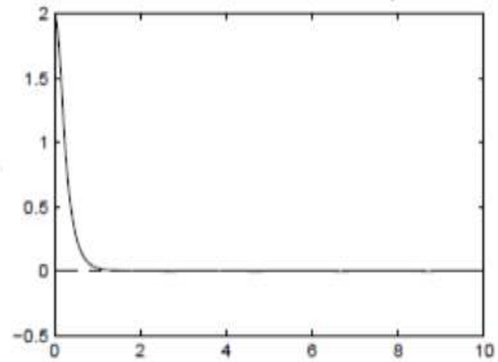
constants were chosen to be c = 5 and ρ = 50. The next plot in Figure 1.2 clearly illustrates the reaching phase and the sliding phase. Although the plot of x_1 shows a good response the plot of u illustrates one of the major drawbacks of variable structure control systems, namely chattering. Methods of overcoming this problem are discussed in the remainder of this chapter.

Plot of the First Variable x_1



Time in Sec.

$\dot{x}_1 = x_2, \dot{x}_2 = f(x) + g(x)\,u + \delta(t),$

$u = u_{eq} - k \cdot sign(s)$

$f(x) = x_1^2 + \sin(x_2)$ and $\delta(t) = \sin(t)$

with, c = 5 and ρ = 50

Fig.1.1

Phase Plane Plot $x_1 and x_2$

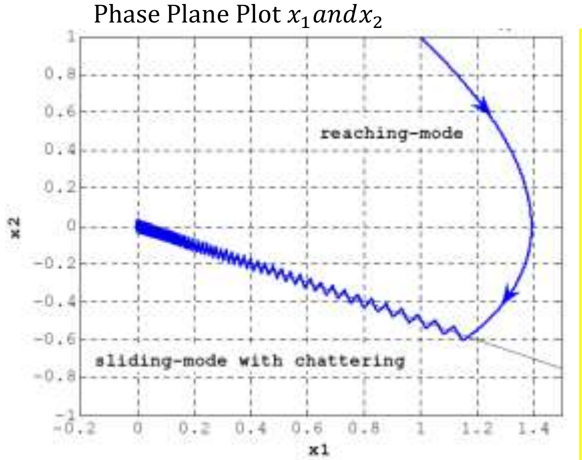


Fig.1.2 Dotted line s = 0, the Sliding Line

2.4 Chattering

One important drawback of the ideal sliding mode controller in Eqn.2.5 is that it leads to a discontinuous control law which in any practical implementation of the controller will result in a phenomenon known as chattering[2].

As described by Young (2002) there are two mechanisms that produce chattering:

• The ideal switching required to implement the controller in (1.11) implies an infinite switching frequency that is impossible to achieve in a practical implementation of the controller.

• Parasitic dynamics in sensors and actuators that produce low amplitude, high frequency oscillation in the neighborhood of the switching manifold.

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2.4.1. Chattering Avoidance: Attenuation and Elimination

In many practical control systems, including DC motors and aircraft control, it is important to avoid control chattering by providing continuous/smooth control signals: for instance, aircraft aerodynamic surfaces cannot move back and forth with high frequency, but at the same time it is desirable to retain the robustness/ insensitivity of the control system to bounded model uncertainties and external disturbances.

3.1 Chattering Elimination: Quasi-Sliding Mode (Boundary layers solution)

One obvious solution to make the control function (1.11) continuous/smooth is to approximate the discontinuous function $v(\sigma) = -\rho \text{ sign}(\sigma)$ by some continuous/ smooth function. For instance, it could be replaced by a "sigmoid function"

$$\text{sign}(\sigma) \approx \frac{\sigma}{|\sigma|+\epsilon} \qquad (1.12)$$

where ϵ is a small positive scalar. It can be observed that point-wise

$$\lim_{\epsilon \to 0} \frac{\sigma}{|\sigma|+\epsilon} = \text{sign}(\sigma)$$

for $\sigma \neq 0$. The value of ϵ should be selected to trade off the requirement to maintain an ideal performance with that of ensuring a smooth control action. The sigmoid function (1.12) is shown in Fig. 1.2.

(1) Ambrosino's app. (2) arctangent
(3) hyperbolic tangent

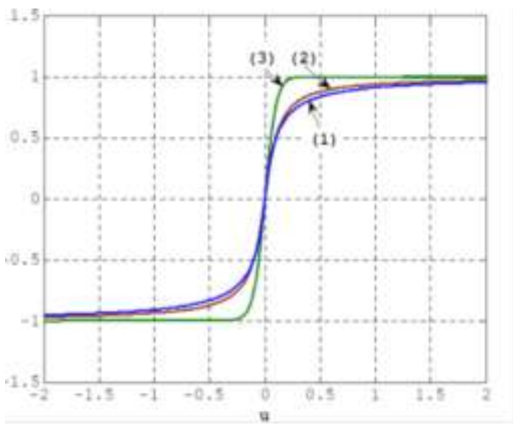

Fig.1.2. Smooth approximation functions to enforce pseudo-sliding-mode with boundary layers

(s/φ) where (1) φ =0.1 (2) φ =0.5 (3) φ = 1.0

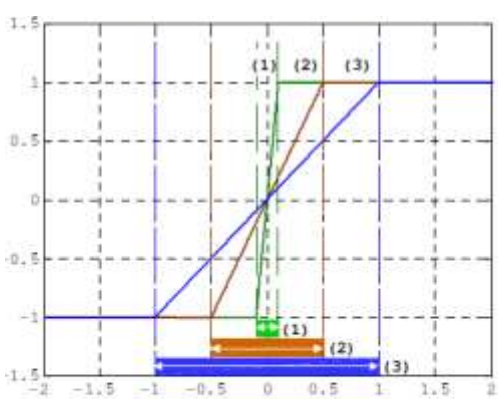

Fig.1.5. saturation functions with various slopes for different bandwidths

The hard-switching sign function in the control signal u described by (1.11) is replaced with the saturation function:

$$u = -cx_2 - \rho \,\mathrm{sat}\left(\frac{s}{\varphi}\right) = -cx_2 - u_{sw} \qquad u_{sw} = \begin{cases} \frac{s}{\varphi} & \text{if } |s| \leq 0 \\ \mathrm{sign}(s) & \text{if } |s| \geq 0 \end{cases} \tag{1.13}$$

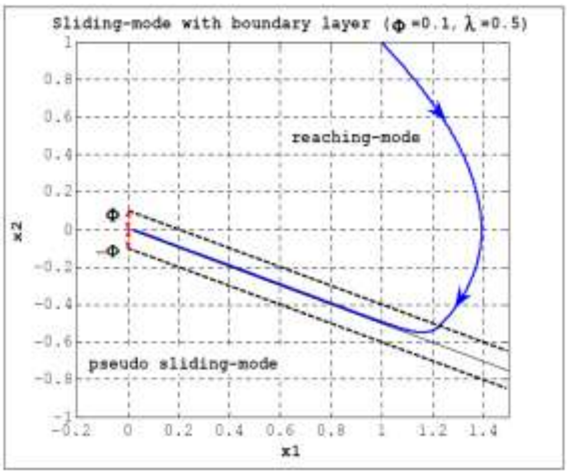

Fig.1.3. Pseudo-Sliding-Mode Control with boundary layers of the plant described by (1.13)

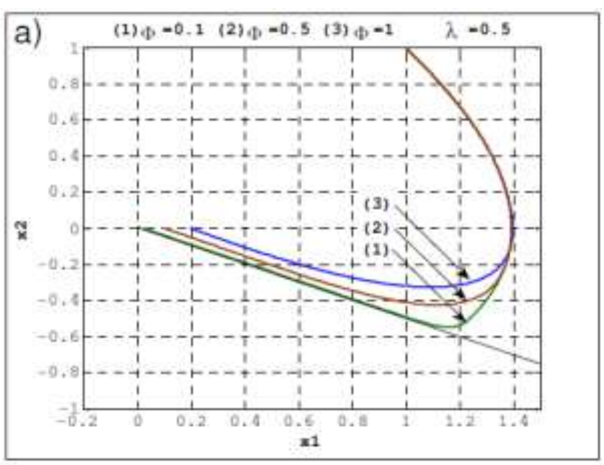

Fig.1.4. a) system trajectories in sliding-mode with boundary layers for different F values;

The two-sided region of the total bandwidth 2·F alongside the switching line **s** = 0 shown in Fig.1.3. becomes the domain of attraction avoiding the control chatter. Hence inside the boundary layers the state trajectory moves towards the origin asymptotically without crossing the switching line at any time instant.

This phase is called *pseudo-sliding-mode* or *quasi-sliding-mode* since the switching action is replaced by a continuous approximation. However, since in case of the boundary layer solution the asymptotic stability is provided, this is obtained at the cost of the increased tracking error. Fig.1.4. presents the state trajectories of the plant (1.1) for different values of the width F of the boundary layers and for the slope φ = 0.5. The corresponding saturation functions enforcing pseudo-sliding-mode have been shown in Fig.1.5. It can be noticed that for higher values of F the tracking error x_1 becomes unacceptable.

The price we pay for obtaining a smooth control function is a loss of robustness and, as a result, a loss of accuracy. The designed smooth control (1.13) is technically not a sliding mode control and there is no ideal sliding mode in the system (1.1), since the sliding variable has not been driven to zero in a finite time. However, the system's performance under the smooth control law in Eq. (1.13) is close to the system's performance under the discontinuous sliding mode control (1.19). This gives us grounds for calling the smooth control law in Eq. (1.13) a *quasi-sliding mode control* and the system's motion, when the sliding surface converges to a close vicinity of the origin, a *quasi-sliding mode*.

3.1.1. Limitations of the Boundary Layer Technique

Although the boundary layer approach to constructing a practical sliding mode controller looks very promising at first, mainly because of its simplicity, there are many limitations that prevent it from being the optimal solution to the chattering problem. As Young (2002) points out, the piecewise linear approximation of the switching control actually reduces the closed loop system into a system without sliding mode. Another important limitation of this technique is that it compromises the robustness and disturbance rejection properties that are actually the biggest strengths of Sliding Mode Controllers.

3.2 Chattering Attenuation: Asymptotic Sliding Mode

In this section we consider another approach to designing continuous control that is robust to bounded disturbances. The idea is to design an SMC in terms of the control function derivative. In this case the actual control, which is the integral of the high-frequency switching function, is continuous. This approach is called chattering attenuation, since some periodic residual is observed in the sliding mode control after the integration of the high-frequency switching function.

Example 2.

To proceed, the system in Eq. (1.1) is rewritten as

$$\begin{cases} \dot{x}_1 = x_2 & x_1(0)=x_{10} \\ \dot{x}_2 = u + f(x_1, x_2, t) & x_2(0)=x_{20} \\ \dot{u} = v & u(0)=0 \end{cases} \quad (1.14)$$

We know that if the sliding variable **s** is constrained to zero in finite time t = tr, then the state variables converge to zero asymptotically in accordance with Eq. (1.14) for all t ≥ tr . Here we assume $|f(x_1, x_2, t)| \le L$ and in addition that it is smooth with bounded derivative

$|\dot{f}(x_1, x_2, t)| \le \bar{L}$.

In order to achieve chattering attenuation the following auxiliary sliding variable

$$s = \dot{\sigma} + \bar{c}\,\sigma \quad (1.15)$$

is introduced. If we design a control law v that provides finite-time convergence of s → 0, then the ideal sliding mode occurs in the sliding surface

$$s = \dot{\sigma} + \bar{c}\,\sigma = 0 \quad (1.16)$$

and σ, $\dot{\sigma} \to 0$ together with x_1, $x_2 \to 0$, as time increases, even in the presence of the bounded disturbance f (x_1, x_2, t). However, we will not have an ideal sliding mode, but instead an asymptotic sliding mode will occur in system (1.14) since the original sliding variable σ converges to zero only asymptotically. This is the price we are going to pay for the chattering attenuation. Using Eq. (1.6) and (1.16) for designing the SMC in terms of v, we obtain

$$s\,\dot{s} = s\,(v + c\bar{c}\,x_2 + (c + \bar{c})\,u + (c+\bar{c})\,f(x_1, x_2, t) + \dot{f}(x_1, x_2, t) \quad (1.17)$$

Choosing $v = -c\bar{c}\,x_2 - (c + \bar{c})\,u + v_1$ and substituting it into (1.17), we obtain

$$s\,\dot{s} = s\,(v_1 + (c + \bar{c})\,f(x_1, x_2, t) + \dot{f}(x_1, x_2, t)) \le s\,v_1 + |s|(\bar{L} + (c + \bar{c})\,L) \quad (1.18)$$

Selecting $v_1 = \rho\,\mathrm{sign}(s)$ with $\rho > 0$, and substituting it into Eq. (1.18) it follows that

$$s\,\dot{s} \le |s|(-\rho + \bar{L} + (c + \bar{c})\,L) = -\frac{\alpha}{\sqrt{2}}|s| \quad (1.19)$$

Finally, if the control gain ρ is computed as

$$\rho = \bar{L} + (c + \bar{c})\,L + \frac{\alpha}{\sqrt{2}} \quad (1.20)$$

then the control law v that drives s to zero in finite time $t_r = \frac{\sqrt{2}|s(0)|}{\alpha}$ is

$$v = -c\bar{c}\,x_2 - (c + \bar{c})\,u - \rho\,\mathrm{sign}(s) \quad (1.21)$$

The results of the simulation of system (1.14) with the sliding mode control (1.15), (1.21), the initial conditions $x_1(0) = 1$, $x_2(0) = -2$, the control gain $\rho = 30$, the parameters c = 1.5; $\bar{c} = 10$, and the disturbance f (x_1, x_2, t) = sin (2t) (which, again, is only used for simulation purposes) are presented in Figs. 1.6–1.8.

The control law (1.21) contains the high-frequency switching term ρ sign(s) that yields chattering (Fig. 1.6); however, chattering is attenuated in the physical control $u = \int v dt$ (Fig. 1.7). It can be observed that the auxiliary sliding variable s converges to zero in finite time and the original sliding variable σ converges to zero asymptotically. Therefore, the achieved sliding mode is called an *asymptotic sliding mode* with respect to the original sliding variable σ .

The state variables exhibit convergence to zero as time increases (see Fig. 1.8) in a similar way to the results of ideal control, that were achieved via the high frequency switching sliding mode control u given by Eq. (1.11). Also, in order to implement the continuous sliding mode control $u = \int v dt$, with v given by Eqs. (1.15) and (1.21), it is necessary to differentiate σ. In the example, $\dot{\sigma}$, was computed numerically; however, it can be done using the sliding mode observers/differentiators that will be discussed later on.

Fig.1.6. Control Law 1.21's ρsign(s) part produces Chattering

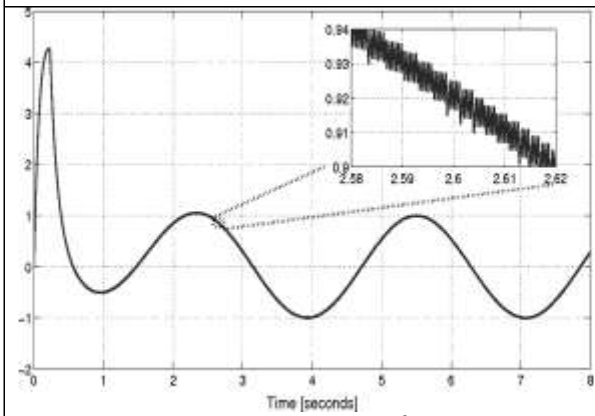

Fig.1.7. Chattering attenuated by u = $\int v dt$

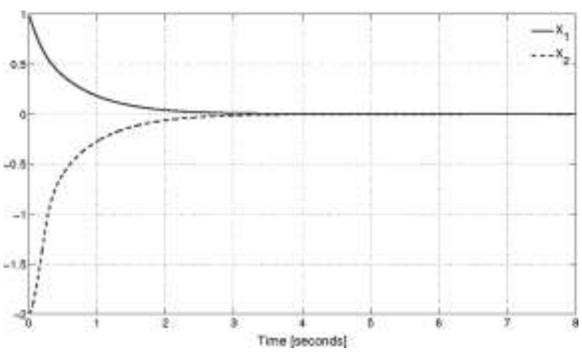

Fig.1.8.Convergence of State variables

4. The Matching Condition and Insensitivity Properties

It was discussed in example.1 that the system's dynamics in the sliding mode do not depend on the bounded disturbance f(x_1, x_2, t). We need to bear in mind that the disturbance f(x_1, x_2 t) enters only the second equation of the system (1.1). The question is whether this *insensitivity property* of the system's dynamics in the sliding mode to the bounded disturbances/uncertainties can be extended to bounded disturbances/uncertainties entering the first equation of the system (1.1).

In order to address this issue consider the system

$$\begin{cases} \dot{x}_1 = x_2 + \varphi(x,t) & x_1(0) = x_{10} \\ \dot{x}_2 = g(x,t)\,u + f(x,t) & x_2(0) = x_{20} \end{cases} \quad (1.22)$$

where $|\varphi(x1; x2;t)| \le P$. Assume that an SMC u is designed to drive the trajectories of system (1.22) to the sliding surface $\sigma = x_1 + cx_2 = 0$ in finite time t ≥ tr and to maintain motion on the surface thereafter. The dynamics of system in the sliding mode can be easily shown by reduced-order motion is described by

$$\begin{cases} \dot{x}_1 = x_2 + \varphi(x_1, x_2, t) \\ \dot{x}_2 = -cx_1 \end{cases} \quad x_1(t_r) = x_{1r} \quad (1.23)$$

It can be observed from Eqs. (1.22)–(1.23) that the disturbance f(x_1, x_2, t) does not affect the system's dynamics in the sliding mode while the disturbance φ(x_1, x_2, t) that enters the first equation (where the control is absent) can prevent the state variable from converging to zero in the sliding mode of system (1.22). The disturbance $f(x_1, x_2, t)$ is called a disturbance *matched* by the control, and the disturbance $\varphi(x_1, x_2, t)$ is called an *unmatched* one.

Note that such a criterion for detecting *matched* and *unmatched* disturbances is valid only for SISO systems with the control u entering in only one equation. The matching condition is to be generalized further on for nonlinear systems of an arbitrary order.

5. Conclusion: Boundary layer approach, eliminates chattering at the cost of robustness and disturbance rejection. Asymptotic Sliding Mode, shift the chattering outside the actuators and retains robustness and disturbance rejection. Other methods of chattering eliminations available in literature are given below:

- replacing the boundary layer with a sliding sector (Shyu et al., 1992; Xu et al., 1996);
- using dynamic sliding mode controllers (Sira-Ramirez, 1993a; Sira-Ramirez, 1993b; Zlateva, 1996);
- using second (or higher) order sliding mode controllers (Bartolini et al., 1998; Levant, 1993).

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